The Synopsis For the Registration of Ph.D. Degree in MATHEMATICS

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Entitled

"Some Studies In Non-linear Integral Equations and Its Application"

By

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1. Abstract of the proposed work plan/Problem:

In this research work, we will study some existing method of solving nonlinear integral equations using classical calculus and fractional calculus. We will try to modify some of these methods and compare it with existing method of solving nonlinear integral equations. We will use different mathematical software to solve nonlinear integral equations.

2) Introduction:

It is well known that Linear and nonlinear integral equations arises in many scientific field such as the population dynamics, spread of epidemics, and semiconductor devices. There are various methods to solve nonlinear integral equations. Here we discuss some of them for nonlinear Volterra integral equations. Same will be applicable to nonlinear Fredholm integral equation. The nonlinear volterra integral equations of the second kind is represented by the form

$$u(x) = f(x) + \int_0^x K(x,t)\, F\big(u(t)\big)dt \ \ldots\ldots\ldots\ldots (1)$$

The nonlinear volterra integral equations of the second kind is represented by the form

$$f(x) = \int_o^x K(x,t)\, F\big(u(t)\big)dt$$

For these two kind of equations,the kernel $K(x,t)$ and the function $f(x)$ are given real valued functions and $F\big(u(x)\big)$ is a nonlinear function of $u(x)$ such as $u^2(x), \sin\big(u(x)\big), and\ e^{u(x)}$.

i) The successive Approximations Method:

This method solves any problem by finding successive approximations to the solution by starting with an initial guess, called zeroth approximation. The successive approximations method introduce the recurrence relation

$$u_{n+1}(x) = f(x) + \int_0^x K(x,t)\, F\big(u_n(t)\big)dt, n \geq 0.$$

We always start with an initial guess for $u_o(x)$. Mostly we select 0,1,or x for $u_o(x)$. Consequently solution $u(x)$ is obtained by using

$$u(x) = \lim_{n\to\infty} u_n\,(x)$$

ii) The Series Solution Method:

In this method, we assume that solution $u(x)$ of nonlinear volterra integral equation is analytic and therefore it posses Taylor series at $x = 0$ which can be written as

$$u(x) = \sum_{n=0}^{\infty} a_n x^n \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

We put this series expression in (1).Where the coefficient a_n will be determined recurrently. Having determined the coefficient $a_j, j \geq 0$,the series solution follows immediately upon substituting the derived coefficient into (2). The exact solution may be obtained if such an exact solution exists. If exist solution is not obtainable, and then the obtained series solution can be used for numerical purposes.

iii) The Adomian Decomposition Method :

In this method, we express nonlinear term $F(u(x))$ into Adomian polynomials. General formula for Adomian polynomial is given by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right]_{\lambda=0} \quad , n = 0,1,2 \dots \dots \dots$$

3) Origin of research problems:

While searching for research problems, we come across some research papers in which researchers used various method to solve nonlinear integral equation like Adomian decomposition method, VIM, Successive approximation method, Weighted Mean Value Theorem etc. These motivate us to go further in that direction as nonlinear integral equations plays important role in physics, population dynamics, spread of epidemics, and semiconductor devices.

4) Preview of literature and development in the subject (Previous work done in the related area):

In 2008, Ghorbani & Saberi-Nadjafi explained how to solve nonlinear integral equations by a modified homotopy method. For more details see[10]. In 2012, Majid khan, M.A.Gondal and sunil kumar published a research paper on A new analytical solution procedure for for solving a nonlinear integral equation. In this paper, they explained two step Laplace decomposition method for nonlinear integral equations. For more details regarding this refer [9]. In 2016, Ahmet Alturk published research paper on neumerial solution of linear and nonlinear integral equations by using

weighted mean value theorem. The statement of weighted mean value theorem is given by

$Let\ \phi\ \psi: [a,b] \to R\ be\ continous\ function\ on\ [a,b]. If\ \psi$

$never\ changes\ sign\ in\ [a,b], then\ there\ exists\ a\ number$

$c \in [a,b]\ such\ that$

$$\int_a^b \phi(x)\psi(x)dx = \phi(c)\int_a^b \psi(x)dx$$

In that research paper, they have given use of weighted mean value theorem for following class of Fredholm integral equations :

- Linear and nonlinear Fredholm integral equation of second kind.
- Linear and nonlinear Fredholm integro–differential equations of the second kind.
- Linear and nonlinear system of Fredholm integral equations of the second kind.
- Linear and nonlinear system of Fredholm integro differential equations of the second kind.

For more details refer [8].

For more details regarding Preview literature and development the subject see references.

(5) Objective of research/Proposed Hypothesis:

To study various method available so far to find solution of nonlinear integral equation and to extend or modify some of these method to get new method of solving nonlinear integral equations. Also to compare existing methods of solving integral equations.

(6) Methodology to be adopted:

i) Initially for review purpose, some research papers, Journals, books, articles will be collected from internet and various institutions. To participate in various national as well as international conferences, seminars and workshops. To study various existing method to solve nonlinear integral equations. To use various mathematical software like MATLAB, MAPLE. Present outcome in various international and national conferences, workshops and seminars**.**

(7) Importance of study / Society applications:

a) Interdisciplinary Relevance:

i) Integral equations arise in many scientific and engineering problems. A large class of initial and boundary value problems can be converted to Volterra and Fredholm integral equations.

ii) The potential theory contributed more than any field to give rise to integral equations. Mathematical physics model such as diffraction problems, scattering in quantum mechanics, conformal mapping, and water waves also contributed to the creation of integral equations.

iii) Our results will save time and will get accurate solution when we solve integral equations.

b) International and National Status:

i) Some researchers from national and international level are solving nonlinear integral equations using various method and mathematical softwares like MATLAB, MAPLE etc. M.A.Hernandez-Veron, Sonia Yadav, Eulalia Martinez, Suk hjit Singh, from Iran Solved nonlinear integral equations with non-seperable kernel via a higher order iterative process. Ahemt Alturk from Turkey published paper on numerical solution of linear and nonlinear Fredholm integral equations using weighted mean value theorem. Majid Khan, Muhammed Asif Gondal, Sunil Kumar from Pakistan & India developed a new analytical solution procedure for nonlinear integrals. For more details refer references given at the end of this synopsis.

c) Significance of the study:

Non-linear integral equation is very useful in physics, Engineering, quantum mechanics, differential equations, etc. Use of different mathematical technique will save time and provide accurate solution.

8)Proposed work plan/Formulation and structure of the study:

Ist Year plan

i) In this year, we will collect basic material required for this research. It includes collection of Books, Journals, and Research Papers.

ii) For this purpose, we will visit to few libraries and institutions like IIT, TIFR, IIS, ISI, and CMI and Some state & Central universities for the collection of literature.

iii) To learn more advanced version of some Mathematical Software.

iv) Also in the same year, we will complete course work.

v) To attend the seminars, Conferences, Workshops on relevant topic.

vi) Two Progress report will be submitted at the end of this year.

IInd Year plan

i) To study collected materials.

ii) To attend the conferences/Workshops/Seminars on relevant topic.

iii) To formulate new problems on relevant topic.

iv) To study existing technique. New technique will be applied on these problems.

v) To discover finding in the direction of research problem.

vi) To develop new technique related to research problem.

vii) To try to publish some research papers on related research topic.

viii) To submit two Progress report to the university.

IIIrd Year plan

i) To Study application of obtained results.

ii) To present obtained outcome in International/National gathering such as Conference, Seminar, Workshop etc

iii) To publish some research papers in reputed journals

iv) To submit two progress reports to university.

v) To submit the work to the University.

References

[1] **Abdul-Majid Wazwaz,** Linear and Nonlinear Integral Equation, Method and Application, Higher education press, Beijing (2011).

[2] **M.A.Hernandez-Veron, Sonia Yadav, Eulalia Martinez, Sukhjit Singh**, Solving nonlinear integral equations with non-seperable kernel via a higher order iterative process,Applied Mathematics & Computation (ELSEVIER) 2021.

[3] **Ahemd A.Khidir**, A New Numerical Technique for Solving Volterra Integral Equations Using Chebyshev Spectral Method, Mathematical Problem in Engineering (Hindawi) 2021

[4] **Hosseian Jafari**, A new general integral transform for solving integral equations, Journal of Advanced Research (ELSEVIER) 2020.

[5] **Farshid Mirzaee,Sahar Alipour**, Approximate solution of nonlinear quadratic integral equations of fraction order via piecewise linear function, Journal of Computational and Applied Mathematics (ELSEVIER) 2017.

[6] **Ahemd A.Hamoud, Kirtiwant P.Ghadle ,** The Reliable modified of Laplace decomposition method to solve nonlinear integral Volterra-Fredholm integral equations.

[7] **Mohamed A.Ramadan and Mohamed R.Ali** ,An efficient hybrid method for solving Fredholm integral equations using triangular functions. New Trend in Mathematical Sciences (BISKA) 2017.

[8] **Ahemt Alturk,** Numerical Solution of linear and nonlinear Fredholm integral equations using weighted mean value theorem, (springer plus) 2016.

[9] **Majid Khan, Muhammed Asif Gondal, Sunil Kumar**, A new analytical solution procedure for nonlinear integral equations. Mathematical and Computing Modelling (ELSEVIER) 2012.

[10] **Asghar Ghorbani , Jafar Saberi-Nadjafi** , Exact Solution for Nonlinear integral equations by modified homotopy perturbation method. International journal of Computer and Mathematics with Application 2008.

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